Spatial dynamics and optimal resource extraction

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Abstract

Extraction from a common pool resource may result in a divergence between competitive and optimal rates of extraction. This paper develops a theoretical model to estimate the size of the payoffs from this divergence under alternative spatial representations. Results show that when a resource is heterogeneously distributed spatially, assuming a spatially homogeneous distribution can underestimate the losses with competitive extraction. An application of the model to a real-world aquifer shows the importance of recognizing spatial heterogeneity in resource extraction problems to: (1) provide robust estimates of the costs of sub-optimal extraction and; (2) implement appropriate corrective policies.

Keywords: spatial dynamics; common pool resources; externalities; groundwater extraction. 

JEL Classification: Q20, Q25, Q30.

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1. Introduction

A central issue in resource economics is estimating the magnitude of the welfare loss from competitive extraction of common pool resources. Although considerable research has been devoted to this issue, studies have neglected to consider the robustness of their results to heterogeneous spatial representations of the resource. In this article, we compare optimal and competitive extraction paths under alternative hypotheses on the distribution of resources over space. The estimated gap between these two scenarios is likely to be incorrect if the dynamics of a heterogeneously distributed resource are represented in a homogeneous specification. Indeed, a precise estimate of the size of the loss from competitive extraction is critical to delineate the appropriate size and scope of resource management policy.

A historical knowledge paradigm in resource economics has depicted resources as homogeneously distributed in space (Sanchirico and Wilen 2005). With the advancement of spatial data generation techniques and interdisciplinary research efforts, recent studies have incorporated a certain degree of spatial heterogeneity in their models (Bockstael 1996; Brozovic et al 2006; Chakraborty, Hochman and Zilberman 1995; Gaudet, Moreaux and Salant 2001; Goetz and Zilberman 2000; Knapp and Schwabe 2008; Parker 2007; Smith, Sanchirico and Wilen 2009). However, the sensitivity of results from temporal resource allocation models to alternative spatial representations remains unexplored.

To address this gap in the existing literature, we build a theoretical model to compare optimal and competitive extraction paths of spatially distributed users yielded by two different spatial representations of the resource. The first representation resembles the most commonly used by economic studies. This specification assumes that the resource is spatially homogeneous
and evolves independently of the history of past extractions. The alternative representation relaxes these standard restrictions and allows for a heterogeneous distribution of the resource, and lagged effects of past extractions.

We find that derived conditions for optimal extraction paths are likely to be incorrect if a heterogeneously distributed resource is depicted by a homogeneous spatial representation. Further, the estimated divergence between optimal and competitive extraction paths may be inaccurate, particularly when the number of users of the resource is increasing over time. We also examine these findings in an empirical groundwater application. Our results show that a homogeneous ‘bath-tub’ representation of groundwater flow, as used by the majority of economic studies on groundwater allocation, fails to capture well-interference areas, thus underestimating the welfare and hydrological costs from competitive extraction.

Overall, our results suggest that assumptions about homogeneity of resource distribution are particularly important in terms of the role and scope of corrective policies on the extraction of common-pool resources. The sensitivity of policies to the spatial characteristics of the resource is important for a wide range of environmental problems represented by a spatial-dynamic process.

This paper is organized as follows. In the following section, we present a theoretical model for optimal dynamic resource extraction by multiple spatially distributed users under two representations of the spatial distribution of the resource. We discuss and compare optimal and competitive extraction conditions from this model under each spatial representation. In the third section, we describe spatial representations in existing economic models of groundwater, while in the fourth section we illustrate the theoretical model with real data
from the Guarani Aquifer System (GAS). We demonstrate how spatial homogeneity hypotheses on the distribution of resources can affect optimal allocation rules and policy prescriptions. The final section concludes by discussing implications for resource management policy and further research effort.

2. The Model

A theoretical model of optimal dynamic resource extraction is presented under two spatial representations of the dynamics of the resource’s response: uniform and immediate (UI) and non-uniform and lagged (NUL). We analyze the case of an exogenously growing demand. Instead of assuming a specific growth structure (Brill and Burness 1994; Brown and Deacon 1972), we represent growth as a given per-period rise in the number of users. Hence, the problem becomes one of choosing both optimal extraction paths and locations for new users.

Consider a resource initially exploited by \( N \) spatially distributed users. In any period, the number of users increases by an exogenous number \( n_t \), conforming a total of \( n(t) \) users. A new user \( i \) at time \( t \) is located at an endogenously decided two-dimensional point \((r_{i1}, r_{i2}) \in \Omega(t)\), where \( \Omega(t) \) is an exogenously bounded feasible region. The decision variable \( u_{it} \) is user \( i \)'s per-period resource extraction at time \( t \). The state variable \( y_{it} \) is defined as the resource stock for user \( i \) at time \( t \). The net benefit from resource extraction is given by the function \( G_i(y_{it}, u_{it}) \) for each initial user and \( G_i(y_{it}, u_{it}, (r_{i1}, r_{i2})) \) for each new user. Note that this function is allowed to vary across users and time. We assume \( \frac{\partial G_i(\cdot)}{\partial y_{it}} > 0 \) and \( \frac{\partial G_i(\cdot)}{\partial u_{it}} > 0 \), so that per-period net benefits increase with both stock and extraction levels. \( y_{it}^{*} \) and \( u_{it}^{*} \) are time dependent constraints on minimum values of stock and use rates.
The UI and NUL sub-models determine specific equations of motion describing the evolution of the state variable over time for every user. According to the UI representation, the resource stock responds to extraction in a temporally and spatially homogeneous fashion. The per-period stock for user \( i \) at time \( t+1 \) \( (y_{i,t+1}) \) is given by the function 
\[
 f_i \left( \sum_{k=1}^{t} \sum_{j=1}^{n(t)} u_{jk} \cdot y_{ij}, w \right)
\]
where \( w \) is the deterministic and constant rate of recharge of the resource. At any time period, stock changes by the same amount for every user. Additionally, variations in extraction rates at \( t \) have an effect only on stock changes from \( t \) to \( t+1 \) which is independent of the location of extraction. Hence, 
\[
 \frac{\partial (f_i[j] - f_{i,t-1}[i])}{\partial u_{jm}} = 0 \forall m = 1, ..., t - 1 \quad \text{and} \quad \frac{\partial f_i}{\partial u_{ik}} = \frac{\partial f_i}{\partial u_{jm}} = \frac{\partial f}{\partial u} < 0 \quad \text{for any pair of users} \; j \; \text{and} \; i \; \text{and} \; k, m \leq t.
\]

The NUL representation is more realistic because it assumes that the resource stock is spatially heterogeneous and dependent upon the complete history of extraction. User \( i \)'s stock at \( t+1 \) is given by 
\[
 f_i \left[ u_{j1} \left| u_{j2} \right|_{j=1}^{n(t)} \left| y_{i1}, w \right| \right].
\]
The resource dynamics is hereby characterized by four distinctive features. First, stock changes are allowed to be non-uniform across space, that is 
\[
 f_{i,j} - f_{i,j-1} \quad \text{may be different from} \quad f_{i,j} - f_{i,j-1}.
\]
Second, extractions by two users \( j \) and \( i \) may have a different effect on a third user \( k \)'s stock: 
\[
 \frac{\partial f_{ik}[j]}{\partial u_{it}} \neq \frac{\partial f_{ik}[i]}{\partial u_{it}}.
\]
Third, extractions at \( t \) may affect stock changes at \( t+m \) and this effect depends on the temporal and spatial distance between extraction and stock changes: 
\[
 \frac{\partial (f_{ik}[j] - f_{ik-1}[i])}{\partial u_{ij-k}} \neq \frac{\partial (f_{ik}[j] - f_{ik-1}[i])}{\partial u_{ij-l}}
\]
where \( k \neq l \). Fourth, we assume that the impact of a new user \( j \)'s extraction on user \( i \)'s stock depends on its location: 
\[
 \frac{\partial f_{ij}}{\partial r_{ij}} \neq 0.
\]
The deterministic discrete time model consists of the maximization of

\[ J = \sum_{t=1}^{T} \beta_t \left[ \sum_{i=1}^{N} G_i \left( y_{it}, u_{it} \right) + \sum_{i=N+1}^{n(t)} G_i \left( y_{it}, u_{it}, (r_{1t}, r_{2t}) \right) \right] \]

subject to

\[ \beta_t = \frac{1}{(1 + \rho)} \]

\[ n(t) = n(t-1) + n_i, \quad t = 1, \ldots, T \]

\[ n(0) = N \]

\[ (r_{1t}, r_{2t}) \in \Omega(t) \quad i = n(t-1) + 1, \ldots, n(t) ; \quad t = 1, \ldots, T \]

\[ y_{it}^* - y_{it} \leq 0 \quad i = 1, \ldots, n(t) ; \quad t = 2, \ldots, T \]

\[ u_{it}^* - u_{it} \leq 0 \quad i = 1, \ldots, n(t) ; \quad t = 1, \ldots, T \]

\[ y_{it+1} = f_t \left[ \sum_{k=1}^{t} \sum_{j=1}^{n(t)} u_{jk}, y_{ij}, w \right] \quad i = 1, \ldots, n(t) ; \quad t = 1, \ldots, T-1 \quad [UI \ representation] \]

\[ y_{it+1} = f_t \left[ u_{ij}, \left( r_{1i}, r_{2i} \right), \sum_{j=N+1}^{n(t)} y_{ij}, w \right] \quad i = 1, \ldots, n(t) ; \quad t = 1, \ldots, T-1 \quad [NUL \ representation] \]

where \((r_{1i}, r_{2i})\) and \((u_{it})\) are the decision variables, the \(y_{ij}\)'s and \(n_i\)'s are given and \(\rho\) is the time discount rate.

A discrete time model was chosen because it is more realistic than a continuous time calculus of variations approach (Burt 1970) and it generates management policies that can be easily implemented (Culver and Shoemaker 1992). The resource’s equations of motions are usually estimated as a linear set of difference equations. Since in a discrete time formulation these equations become summations, the method of Lagrange multipliers can be used to derive the necessary conditions for the problem defined by (1)-(7) under (8a) or (8b) (Brozovic, Sunding and Zilberman 2006).
The lagrangian expressions under the UI and NUL representations are respectively:

\[(9a)\quad L^{ui} = J + \sum_{t=1}^{T} \sum_{i=1}^{n(t)} \left[ \sum_{j=1}^{n(k)} f_{u_{ij}}[y_{ij}, w] - y_{it} \right] + \sum_{t=1}^{T} \sum_{i=1}^{n(t)} \lambda_{it}^{ui} [y_{it} - y_{it}^*] + \sum_{i=1}^{n(t)} \delta_{it}^{ui} [u_{it} - u_{it}^*] \]

\[(9b)\quad L^{mul} = J + \sum_{t=1}^{T} \sum_{i=1}^{n(t)} \left[ \sum_{j=1}^{n(k)} f_{u_{ij}}[y_{ij}, w] - y_{it} \right] + \sum_{t=1}^{T} \sum_{i=1}^{n(t)} \lambda_{it}^{mul} [y_{it} - y_{it}^*] + \sum_{i=1}^{n(t)} \delta_{it}^{mul} [u_{it} - u_{it}^*] \]

where \( \lambda_{it}^{ui}, \delta_{it}^{ui}, \lambda_{it}^{mul}, \delta_{it}^{mul} \) and \( y_{it}^* \) are the Lagrange multipliers. Some of the necessary conditions for an interior solution are detailed below. Sufficient conditions for global optimality are quasiconcavity of the \( G_{it} \) function and quasiconvexity of the \( f \) function.

\[(10a)\quad \frac{\partial L^{ui}}{\partial y_{it+1}} = 0 \Rightarrow \frac{\partial G_{it+1}(t)}{\partial y_{it+1}} + \nu_{it+1}^{ui} = \mu_{it}^{ui} \beta_{it}^{-1} \quad i = 1, \ldots, n(t) \quad t = 1, \ldots, T-1 \]

\[(10b)\quad \frac{\partial L^{mul}}{\partial y_{it+1}} = 0 \Rightarrow \frac{\partial G_{it+1}(t)}{\partial y_{it+1}} + \nu_{it+1}^{mul} = \mu_{it}^{mul} \beta_{it}^{-1} \quad i = 1, \ldots, n(t) \quad t = 1, \ldots, T-1 \]

\[(11a)\quad \frac{\partial L^{ui}}{\partial u_{it}} = 0 \Rightarrow \frac{\partial G_{it}(t)}{\partial u_{it}} + \psi_{it}^{ui} = -\frac{\partial f}{\partial u} \sum_{k=1}^{T-1} \sum_{j=1}^{n(k)} \mu_{jk}^{ui} \beta_{it-k}^{j-1} \quad i = 1, \ldots, n(t) \quad t = 1, \ldots, T \]

\[(11b)\quad \frac{\partial L^{mul}}{\partial u_{it}} = 0 \Rightarrow \frac{\partial G_{it}(t)}{\partial u_{it}} + \psi_{it}^{mul} = -\sum_{k=1}^{T-1} \sum_{j=1}^{n(k)} \mu_{jk}^{mul} \beta_{it-k}^{j-1} \frac{\partial f_{jk}}{\partial u_{jk}} \quad i = 1, \ldots, n(t) \quad t = 1, \ldots, T \]

If \( i \) is a new user from period \( t \) onwards:

\[(12a)\quad \frac{\partial L^{ui}}{\partial r_{im}} = \sum_{k=1}^{T} \frac{\partial G_{ik}(t)}{\partial r_{im}} \beta_{k} = 0 \quad i = n(t-1)+1, \ldots, n(t) \quad m = 1,2 \]

\[(12b)\quad \frac{\partial L^{mul}}{\partial r_{im}} = \sum_{k=1}^{T} \frac{\partial G_{ik}(t)}{\partial r_{im}} \beta_{k} + \sum_{k=1}^{T-1} \sum_{j=1}^{n(k)} \frac{\partial f_{jk}}{\partial r_{im}} \delta_{jk}^{mul} = 0 \quad i = n(t-1)+1, \ldots, n(t) \quad m = 1,2 \]
In order to state the conditions in current value form, we define the transformations
\[ \lambda_{jk} = \beta_{k} \mu_{jk}, \quad \delta_{jk} = \beta_{k} v_{jk} \quad \text{and} \quad \gamma_{jk} = \beta_{k} \sigma_{jk}. \]

Conditions (10a) and (10b) show that for an optimal allocation, the marginal current value shadow price of the resource stock for each user at time \( t \) must equal the discounted marginal net benefit of a further unit of stock in \( t+1 \). Since the stock level at \( t+1 \) is determined by decisions at \( t \), if the shadow price at \( t \) were larger (smaller), benefits would increase by saving less (more) stock for \( t+1 \). Note that the net marginal benefit of a further unit of stock is conformed by the user’s extra profits and the relaxation of the constraint on the minimum value of the stock if it is binding \( (\nu_{it+1}) \).

Conditions (11a) and (11b) equate the private marginal net benefit of extracting an additional unit of the resource with the discounted future costs of that extraction on all users. If the private net marginal benefits of an extra unit of extraction are constant across time, then both equations indicate an increasing optimal path of extraction. However, lower stocks are likely to increase marginal extraction costs over time. Hence, the shape of the optimal extraction path will depend on the importance of higher marginal extraction costs relative to lower future external costs on all other users.

Condition (11a) in the UI representation implies that the optimal extraction paths of two users with the same net benefits function will be exactly equal. This is because the marginal stock depletion caused on other users is independent of the location of the extraction source. Conversely, under the NUL representation, even if two users share the same net benefits function, the external effects of their extractions are allowed to differ so that their optimal extraction paths are likely to vary. Thus, simplifying the resource’s response to an average
rate $\frac{\partial f}{\partial u}$ is likely to over (under) estimate the optimal extraction paths of users who are relatively more (less) harmful to other users’ stocks, ceteris paribus.

Conditions (12a) and (12b) determine the optimal location of new users. If the stock’s response to extractions is uniform throughout the resource site, only private marginal net benefits of the new user are taken into account. By contrast, if the NUL representation is used, new users are optimally distributed across space only when total marginal net benefits of changing the location of a new user are zero. These net benefits consist of the private benefits accruing to the new user and the social benefits derived from choosing locations that interfere less with other users’ stock levels.

We examine the conditions for (11a), (11b), (12a) and (12b) to yield very similar solutions. When the externality imposed on others by each user’s extraction is not diffusional in nature, the effects of extraction are uniformly and immediately transmitted throughout the whole resource. In this case, $\frac{\partial f}{\partial u,\begin{array}{c} jk \end{array}}$ is virtually unchanged for different values of $(i,t)$ and $(j,k)$ and $\frac{\partial f}{\partial r_{ji}}$ and $\frac{\partial f}{\partial r_{j2}}$ are very close to zero. In such settings, the resource is more akin to a homogeneous physical characterization (such as the UI representation) and few further insights can be obtained from a more complex description of the physics of the natural system (NUL representation).
Competitive extraction

We investigate the effect of the spatial representation of the resource on the estimation of inefficiencies resulting from competitive extraction of the resource. We assume that in the absence of regulation, users commit themselves at the start of the program to a complete time path of extraction that maximizes the present value of their stream of profits given the extraction paths of rival users. Since access to most common property resources is restricted by a series of legal and institutional constraints, the incentive to conserve and recognize the interdependence of extraction paths is limited but not absent from private decision-making.

The necessary conditions characterizing user $i$’s problem under the UI and NUL representations of the resource system are as follows:

\[
(13a) \quad \frac{\partial L_{it}^u}{\partial y_{it+1}} = 0 \Rightarrow \frac{\partial G_{it+1}^u}{\partial y_{it+1}} + \nu_{it+1}^u = \mu_{it}^u \beta^{-1}
\]

\[
(13b) \quad \frac{\partial L_{it}^{mul}}{\partial y_{it+1}} = 0 \Rightarrow \frac{\partial G_{it+1}^{mul}}{\partial y_{it+1}} + \nu_{it+1}^{mul} = \mu_{it}^{mul} \beta^{-1}
\]

\[
(14a) \quad \frac{\partial L_{it}^m}{\partial u_{it}} = 0 \Rightarrow \frac{\partial G_{it}^m}{\partial u_{it}} + \omega_{it}^m = -\frac{\partial f}{\partial u} \sum_{k=1}^{T-1} \mu_{it}^{u} \beta^{-k}
\]

\[
(14b) \quad \frac{\partial L_{it}^{mul}}{\partial u_{it}} = 0 \Rightarrow \frac{\partial G_{it}^{mul}}{\partial u_{it}} + \omega_{it}^{mul} = -\sum_{k=1}^{T-1} \mu_{it}^{mul} \frac{\partial f_{t}}{\partial u_{it}} \beta^{-k}
\]

If $i$ is a new user from period $t$ onwards:

\[
(15a) \quad \frac{\partial L_{im}^u}{\partial r_{im}} = \sum_{k=1}^{T} \frac{\partial G_{ik}^u}{\partial r_{im}} \beta_k = 0 \quad m = 1, 2
\]
(15b) \( \frac{\partial L^{mul}_m}{\partial r_{im}} = \sum_{k=t}^{t-1} \frac{\partial G_{ik}}{\partial r_{im}} \beta_k + \sum_{k=t}^{t-1} \alpha_{ik} \frac{\partial f_{ik}}{\partial r_{im}} = 0 \quad m=1,2 \)

Once again, we define the transformations \( \lambda_{jk} = \beta_k u_{jk} \), \( \gamma_{jk} = \beta_k v_{jk} \) and \( \delta_{jk} = \beta_k v_{jk} \).

Conditions (13a) and (13b) have the same meaning as (10a) and (10b). We consider three cases of externalities. If the resource’s response to each user’s extraction is localized to the immediate vicinity of that user (\( \frac{\partial f_{ik}}{\partial u_{it}} \approx 0 \) and \( \frac{\partial f_{jk}}{\partial r_{it}} \approx 0 \), \( \frac{\partial f_{ik}}{\partial r_{it}} \approx 0 \) for all \( (k,t) \) and \( j \neq i \)), then by comparing (11b) with (14b) and (12b) with (15b), we can conclude that the solutions yielded by the optimal and competitive extraction schemes under the NUL representation will be close. Hence, inefficiencies from competitive allocation are small. However, by comparing (11a) with (14a), we can see that the UI representation may still find an important divergence between the optimal and competitive schemes and overestimate the policy scope for regulation of the resource. The simpler representation of the physical system fails to acknowledge that the resource is close to private property to begin with.

In intermediate cases where externalities are diffusional over space (\( 0 < \frac{\partial f_{ik}}{\partial u_{it}} < \frac{\partial f_{ik}}{\partial u_{it}} \)), the NUL representation concludes that inefficiencies from competitive extraction are important. By contrast, the UI representation under (over) estimates the gap between optimal and competitive extraction paths of users who are relatively more (less) harmful to other users’ stocks so there is a chance that no practical inefficiency is found at all.
Finally, in the case where the effects of extraction are widely transmitted throughout the resource in a uniform fashion \( \frac{\partial f_i}{\partial u_{it}} \approx \frac{\partial f_j}{\partial u_{jt}} \approx \frac{\partial f}{\partial u} \forall i \neq j \text{ and } k < t \), the UI representation is able to adequately capture the behavior of the resource.

Even if in the last two cases the gap estimated between optimal and competitive extraction paths is the same regardless of the spatial representation used, the gains from optimal management will be underestimated by the UI model. This result can be derived by comparing (15a) with (12a) and (15b) with (12b). Since stock depletion at both the own and others’ sites is independent of the location of the extraction source in the UI representation, the optimality conditions for the location of new users only differ between the optimal and competitive scenarios if the NUL representation is used.

**Discretization of spatial aspects of extraction**

The introduction of space in a dynamic optimization framework is more challenging than the consideration of time because of its multiple dimensions. Economic spatial models of resource management have dealt with continuous one-dimensional representations of space that are sufficiently tractable to be analyzed with traditional optimization techniques. These models fix spatial heterogeneity over time (Chakraborty, Hochman and Zilberman 1995; Gaudet, Moreaux and Salant 2001; Goetz and Zilberman 2000; Knapp and Schwabe 2008; Kolstad 1994; Parker 2007; Smith, Sanchirico and Wilen 2009). However, problems with endogenously determined spatial heterogeneity in multiple dimensions cannot be solved analytically (and empirically applied) unless space is discretized.
One way of discretizing the continuous feasible region for the location of users is to convert it into an index set. Next, metamodelling (developing a model of the model) can be used to provide a functional relation between the response (stock changes) and the excitation (extraction pattern) at points where the information has economic value (user locations).

3. Spatial dynamics in groundwater

Groundwater extraction links economic actors over space and time in a spatial-dynamic process that generates spatial-dynamic externalities (Wilen 2007). The spatial structure of these externalities is represented by ‘cones of depression’ where the water table is drawn down in the area adjacent to each pumping well. Where many wells exist, their intersecting cones of depression create complicated patterns in the surface of the groundwater table that evolve through time. Hence, drawdown in any point of an aquifer depends on the location and history of extractions.

Most economic studies of groundwater management use single-cell (or ‘bath-tub’) aquifer models that assume an aquifer responds uniformly and instantly to groundwater extraction. These models suppose that the response of the aquifer depends only upon hydrological parameters and ignore the positioning of development within the system. Various optimal control methods have been applied to derive dynamic optimal groundwater allocations using this simple hydrological representation (Brown and Deacon 1972; Burt 1967 and 1970; Gisser 1983; Koundouri 2004; Tsur and Graham-Tomasi 1991).

A similarly large body of literature has used bath-tub models to measure welfare losses if groundwater allocation is left to the free market. Under quite restrictive economic, hydrologic
and agronomic assumptions, Gisser and Sanchez (1980) found that there is no substantive quantitative difference between optimal rules for pumping water and competitive rates (the Gisser-Sanchez effect (GSE)). Their conclusion has led to a number of investigations on the robustness of the GSE which perform sensitivity analysis on all of its assumptions with the exception of the spatial uniformity of the aquifer. Some of these studies found a significant divergence between optimal and competitive extraction paths (Brill and Burness 1994; Feinerman and Knapp 1983; Kim et al. 1989; Provencher and Burt 1993; Shah, Zilberman and Chakraborty 1995; Worthington et al. 1985) while others did not (Allen and Gisser 1984; Knapp and Olson 1995; Nieswiadomy 1985).

Two-cell models offer a more realistic interpretation of groundwater hydraulics by dividing the simulation region in two cells and allowing flow between them in proportion to the difference in stock levels. Nevertheless, these models only examine interdependency between two areas (such as two adjacent aquifers) and ignore micro-level incentives of individual users within each area (Chakraborty and Umetsu 2003; Saak and Peterson 2007; Zeitouni and Dinar 1997).

A closer approximation to realistic groundwater dynamics has been achieved by a few economic studies that model aquifers as multi-cell basins. These studies represent water movement between cells with finite difference approximations of groundwater flow equations and linearize the system to include it in the economic optimization. The short-coming of these models is that only previous period’s extractions (and not the whole extraction history) are assumed to influence groundwater stock changes at any time period (Culver and Shoemaker 1992; Noel et al. 1980; Noel and Howitt 1982).
Finally, four important contributions to the modeling of spatial heterogeneity and path-dependency in groundwater extraction must be noted. Brozovic, Sunding and Zilberman (2006) built a theoretical model for the optimal extraction of groundwater by spatially distributed users. They conclude that some aquifers may be more akin to private than common property and may be subject to significant lagged effects from pumping. A few decades earlier, Bredehoeft and Young (1970) incorporated spatially dynamic characteristics of aquifer behavior into a simulation program and directly embedded it into an economic optimization problem. However, they only investigated the effects of two policy instruments fixed in space and time. Young, Doubert and Morel-Seytoux (1986) generated response functions to specialized excitations from a finite-difference model and analyzed several institutional alternatives for managing a groundwater-surface water system. Finally, Faisal and Young (1997) used a discrete kernel-based hydrological model to compare socially optimal and open access extraction schemes from a hypothetical basin.

Notwithstanding the contribution to groundwater modeling by the economic literature, none of these studies has addressed the sensitivity of economic results to different spatial and temporal specifications of the aquifer’s response. Moreover and most importantly, no empirical studies have determined in what settings the inefficiencies from competitive extraction will be mismeasured by a homogeneous physical representation in a real-world context.

4. Model Application

The Guarani Aquifer System (GAS) is located in the sedimentary Parana Basin in the subsoil of the east and center-south of South America. As shown by figure 1, we focus on a section
of the aquifer identified as the Concordia-Salto pilot project (The World Bank 2006). The seven wells in the area extract thermal groundwater for balneological purposes. Although access is limited by extraction permits, tourism operators own groundwater as a common property resource subject to the rule of capture. Thus, the rate of groundwater mining and recycling and the location of new wells are the result of private decision-making.

[Insert figure 1 near here]

*The hydrological sub-model: UI and NUL representations*

The hydrologic conceptual model of the local area was developed and parameterized by a study of a Global Environmental Facility (GEF)’s project (Charlesworth, Sangam and Assadi 2008). Following Morel-Seytoux and Daly (1975), the finite difference model is run 50 times by applying different levels of stress at the seven existing and seven potential stress locations. Due to computational constraints, the duration of the study (40 years) is divided in two management/stress periods of 20 years during which extraction rates are held constant. The different stress levels applied in the runs are obtained by randomly generating percentage variations of the actual extraction rates from a normal distribution.²

The three-dimensional flow simulation package MODFLOW is used to simulate the aquifer numerically (McDonald and Harbaugh 1988). After the model is allowed to run in transient mode, drawdowns are recorded at the end of each time interval (20 and 40 years) and well location.

Let \( Q_{1,k} \) and \( Q_{2,k} \) be the extraction rates (in m³/h for a 16-hour daily extraction regime) applied at location \( k \) during the first and second stress periods respectively. Let \( s_{i,1} \) and \( s_{i,2} \)
be the aquifer’s response at location $i$ after 20 and 40 years due to all such stresses. The UI representation entails estimating the following function:

$$(16) \ s_{i,j} = \beta \sum_{k=1}^{14} (Q_{k,j} + Q_{k,j-1}) \ j=1,2$$

Note that the drawdown of the water table is uniform throughout the aquifer and the contribution of each well’s extraction is constant across time and space (the coefficient $\beta$ is constant).

The NUL sub-model is derived by adapting Theis (1946) solution for transient well response to pumping and using the principle of superposition to estimate drawdowns $s_{i,1}$ and $s_{i,2}$ as a linear function of $Q_{k,1}$ and $Q_{k,2}$ $\forall k = 1,..,14$ as:

$$(17) \ s_{i,1} = \sum_{k=1}^{14} Q_{k,1} \beta_{i,k,1}$$

$$(18) \ s_{i,2} = \sum_{k=1}^{14} [Q_{k,1} \beta_{i,k,2} + (Q_{k,2} - Q_{k,1}) \beta_{i,k,1}]$$

The coefficients $\beta_{i,k,1}$ and $\beta_{i,k,2}$ are commonly cited as the ‘well functions’, which depend on the time since extraction started at well $k$ and the distance between wells $i$ and $k$. These represent drawdowns at location $i$ after 20 and 40 years since well $k$ started its extraction, caused by a permanent unit increase in the constant extraction rate applied at location $k$ (Brozovic, Sunding and Zilberman 2006).

Formulation of the socially optimal and competitive management cases
Since users fulfill their demand for water by self-extraction from the aquifer (facing no external price for it), the optimal design policy is derived by converting problem (1) into a cost-minimization problem. The decision variables are (a) where to install two new wells from a set of potential locations, (b) whether to install/de-install a water recycling system at each existent and new location in the first or second period, (c) whether to install/de-install a pumping system at each existent and new location in the first or second period. The constraints are that (a) extraction at each well exceeds a given demand minus the equivalent recycled water (if any), (b) hydraulic heads at all operative well locations must exceed the distance between ground surface and the lower datum of the aquifer by more than 10m if no equipment is installed and more than 3m if water recycling systems but no pumps are installed, and (c) the aquifer’s response to extraction patterns represented by equations (16) or (17) and (18).

In the competitive management scenario, one of the locations for the new wells is given by a well that has already been drilled in the area. The other location is assumed to be selected in a ‘myopic’ fashion based on the largest head excess -expected after the first 20 years- of the distance between ground surface and the lower datum of the aquifer. Two potential sites (one in Argentina and one in Uruguay) are analyzed for the second location.

A decision timeline is depicted on figure 2. All users have perfect information on the equation of motion of hydraulic head at their own wells, and on the water requirements of every other user. New well locations are selected beforehand (and these are public information). Well drilling and construction starts and finishes at time zero. For the first management period, all users assume the rest will extract their total water requirements.
However, ex-post, agents can observe each other’s decisions so that at the start of the second period, they assume that extraction will remain at period one levels.

[Insert figure 2 near here]

The optimization algorithm

Complex groundwater management problems such as optimal reservoir systems operation and remediation/monitoring designs have recently been studied by optimising Monte Carlo techniques such as Genetic Algorithms (GAs), Simulated Annealing and Neural Networks (Hsiao and Chang 2002, McKinney and Lin 1994, Wang and Zheng 1998). This paper uses GA optimization to address the discontinuity of the objective function and the combinatorial nature of the decision variable set.

Data

The empirical application of the present hydro-economic model uses economic and hydrologic data derived from reports published as a result of the aforementioned GEF project (Barbazza 2006; Castagnino 2008; Charlesworth, Sangam and Assadi 2008). Hydrological parameters (surface level, initial hydraulic heads, current extraction rates, etc.) were obtained from the numerical model developed as part of the pilot project in the area (The World Bank 2006).
Simulation results

The coefficients $\beta$, $\beta_{i,k,1}$ and $\beta_{i,k,2}$ from equations (20), (21) and (22) were estimated by linear regressions on data obtained by simulating different extraction scenarios and recording responses at each well.

A graphical analysis of drawdown at one of the wells (well 5) provides evidence of spatial interdependency and the lagged nature of the groundwater externality. The well function analyzed is representative of the response at every other well. Figure 3 presents the impacts through time and space on drawdown at well 5 of a permanent unit change in extraction from each well at the first and the 20th year of the program. Following Brozovic, Sunding and Zilberman (2006), the ‘well function’ is discounted to the time when the extraction rate is incremented.

Initially we consider the effect of distance on the magnitude of the externality imposed by one user on another. A permanent increase in any user’s extraction from the 20th year has a lower impact on well 5 the further the user is from it. However, the relationship between drawdown and the position of extraction diminishes when extraction rates are incremented from the start of the planning period.

This finding brings us to the relevance of lagged groundwater extraction externalities on a path-dependent resource. The NUL representation predicts that at larger distances from an extraction well, the effects of changes in extraction that occurred several periods ago may be more significant than more recent changes, even with discounting.
We can compare the impacts resulting from a change in extraction 20 and 40 years ago of the closest and furthest wells from well 5. If the extraction schedule of one of the three closest wells changes from the initial period, the impact is 96% lower than a more recent change (in the 20th year). However, in the case of one of the three furthest wells the impact is 75% lower. The rich spatial dynamics explained in previous paragraphs cannot be captured by the simple UI representation of the aquifer.

[Insert figure 4 near here]

On figure 4, the solid ‘well function’ is normalized by a user’s own well function. The NUL representation shows that how much a user cares about another user’s groundwater withdrawals depends inversely on the distance between the two. The relative well function decreases rapidly with distance, implying that groundwater users may be concerned only about extraction by only very proximate users. This suggests that this aquifer is an example of an intermediate case where externalities are diffusional over space. It also implies that a spatially uniform policy is most likely to bring few gains over no intervention and perhaps a simple spatial regulation may be more effective.

The UI and NUL representations derive different optimal new well locations as expected by the difference between optimality conditions (12a) and (12b). Under the NUL representation, costs are minimized when the new wells are located at sites #11 and #13 because head and demand constraints are satisfied without the need to invest in any technology in neither the first nor the second management period. Conversely, as figure 5 shows, under the UI representation optimal well locations are sites #9 and #12 and no technology is needed given the drawdown predictions of this physical specification.
The location of new wells in the competitive management scenario is the same regardless of the aquifer’s representation used. At the start of the first management period, all users expect their hydraulic heads to be sufficient to cover their water needs during the period, and no equipment is installed. During the second 20 years, drawdown in the north-eastern corner of the area increases dramatically if measured with the NUL representation. It is worth noticing that this happens regardless of the position of the second new well. Hence, two users in that area are forced to invest in recycling systems. Since the UI representation averages out drawdown throughout the aquifer, it fails to acknowledge the interference area in the north-western corner of the aquifer. As summarized in table 1 and predicted by the theoretical results for diffusional externalities, the UI representation underestimates the welfare losses of a competitive management scheme.

The welfare gains of the optimal policy predicted by the NUL representation are mirrored by hydrologic gains. Panel A of figure 6 shows that in most wells, the competitive management-induced drawdowns are more than five meters greater than the optimal policy-induced drawdowns. By contrast, the UI representation on panel B calculates an equal difference between initial and final heads for every location irrespective of the positioning of new users and management scheme.
5. Conclusions

The magnitude of the welfare loss from competitive extraction of common pool resources has usually been estimated under the assumption that these are homogeneously distributed in space. This article suggests this assumption may not be innocuous. Rather, it may affect the derivation of optimal and competitive extraction paths and the gap between them. We contribute to the discussion of optimal management versus no intervention by providing theoretical and empirical evidence highlighting the importance of more detailed spatial depictions of resource systems to better understand intertemporal externalities.

We develop a theoretical model to compare the divergence between competitive and optimal extraction time-paths and location of new users under alternative spatial representations of the resource. The first representation resembles those most frequently used by economic studies which assume an immediate and uniform response to extraction. By contrast, the alternative and more realistic representation allows for a dynamic spatial heterogeneous response, more akin to the behavior encountered in real-world resources.

Our theoretical model reveals that simplifying heterogeneously distributed resources with homogeneous spatial representations leads to over (under) estimation of optimal extraction paths of users whose extractions are relatively more (less) harmful to others’ stocks. Under the assumption of a spatially homogeneous resource, we find no difference between the conditions for optimal and competitive location of new users and the predicted welfare gap between competitive and optimal extraction will be incorrect if the resource is spatially heterogeneous.
We apply our model to a real-world groundwater for which detailed spatial data is available. Our empirical findings show that significant welfare and hydrologic costs from competitive extraction are overlooked if the aquifer is assumed to be a homogeneously distributed resource. This is because such a representation fails to capture well interference areas. Thus, the location of new wells is an irrelevant decision for users in both the optimal and competitive extraction scenarios.

From a policy perspective, the present study raises important issues. An implication, at least in terms of the aquifer studied, is that second-best economically defined spacing regulations are likely to have better efficiency results than uniform taxes or quotas. While policies that vary idiosyncratically across space may, in some cases, be physically infeasible and/or prohibitively costly to implement, spatially-based policies do offer the potential of higher payoffs than conventional approaches with spatially heterogeneous resources.
References


Figure 1. The Concordia-Salto pilot area.

Figure 2. Decision and installation timeline for the competitive extraction scenario.

Figure 3. Impact of extractions on well #5.
Figure 4. NUL representation: relative impact of extractions on well #5.

(A) NUL representation

(B) UI representation

Figure 5. Optimal new well locations and technology installed.
<table>
<thead>
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<th>Management scheme</th>
<th>Competitive</th>
<th>Optimal</th>
<th>Difference</th>
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<tr>
<td>Location of new wells</td>
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<td>Location of recycling systems</td>
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<td>Location of pumping systems</td>
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<td><strong>B. UI representation</strong></td>
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<tr>
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<td>None</td>
<td>NO</td>
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<td>Location of pumping systems</td>
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<tr>
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<td>Average drawdown after 40 years</td>
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</table>

(A) NUL representation

(B) UI representation

Figure 6. Potentiometric head at active wells at the end of the planning period.
Endnotes

1 Complementary slackness conditions and those assuring that resource dynamic constraints are satisfied have been omitted for brevity.

2 Extraction rates at the seven potential sites for well location were set at 150m$^3$/h for a 16-hour daily extraction regime.

3 The normal distribution used had a mean of 0.5 and a standard deviation of 0.3, as users in the area agreed on the likelihood of an increasing extraction pattern over the next years.

4 The principle of superposition means that for linear systems, the solution to a problem involving multiple inputs (or stresses) is equal to the sum of the solutions to a set of simpler individual problems that form the composite problem.

5 The locations used are the ones proposed by Charlesworth, Sangam and Assidi (2008).

6 Further details are available from the author upon request.

7 Idem.